# Matrix algebra – addition and multiplication

**Exercise 2.12**

Find all matrices $B$ that commute with:

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

in the sense that $AB = BA$

**Solution Exercise 2.12**

The general system of equations has to hold for $A$ and $B$ to commute

$$\begin{cases} a_{11}b_{11} + a_{12}b_{21} = b_{11}a_{11} + b_{12}a_{21} \\ a_{11}b_{12} + a_{12}b_{22} = b_{11}a_{12} + b_{12}a_{22} \\ a_{21}b_{11} + a_{22}b_{21} = b_{21}a_{11} + b_{22}a_{21} \\ a_{21}b_{12} + a_{22}b_{22} = b_{21}a_{12} + b_{22}a_{22} \end{cases}$$

$$\begin{cases} b_{11} + 2b_{21} = b_{11} + 2b_{12} \\ b_{12} + 2b_{22} = 2b_{11} + 3b_{12} \\ 2b_{11} + 3b_{21} = b_{21} + 2b_{22} \\ 2b_{12} + 3b_{22} = 2b_{21} + 3b_{22} \end{cases}$$

$$=$$

$$\begin{cases} b_{21} = b_{12} \\ b_{22} = b_{11} + b_{12} \\ b_{11} = -b_{21} + b_{22} \\ b_{12} = b_{21} \end{cases}$$

$$v = b_{11}, u = b_{12}$$

$$\Rightarrow B = \begin{pmatrix} v & u \\ u & v + u \end{pmatrix}$$